**Relativistic Momentum & Energy**

In the next file, I basically just ‘say’ that p = mvγ. I would like to be more persuasive…So here’s an approach I read on internet (*Sonego* and *Pinn*). They say, start with energy conservation for collision. Since it must hold in all inertial reference frames, make a boost by u and then we have an equation which must hold identically for every u. So take a derivative w/r to u and we will get another conservation equation.



We define the conserved quantity: momentum.



Note that the boost is actually in the negative direction to make this work out. In classical mechanics this works out as following:



But we can derive an expression for p therefore if we know E. But it turns out that we don’t even need to know E, as long as we know the velocity composition law, and some basic relations between p and E. So start with work-energy and impulse-momentum equations:



The ratio of these equations is:



Now going back to definition of p,



We now substitute in what dE is:



and so:



Now we look at the velocity transformation law. First we’ll take a look at the Galilean one: v(u) = v + u. So then, dv/du = 1. And the integral will give us,



where we have, in the classical sense, identified the constant eK as mass m. So then we for the Lorentzian transformation we have:



So filling this into our expression for p we get:



In order that the expression reduce to the classical expectation when v → 0, then we must have eK = m again. So we have:



and then we can go ahead and derive the energy based off of one of those differentials:



Integrate by parts:



and so we have energy to be:



Might observe we have a nice relationship between energy and momentum,



And can also write,



Since this is equivalent to:

