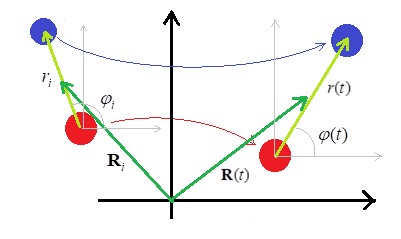
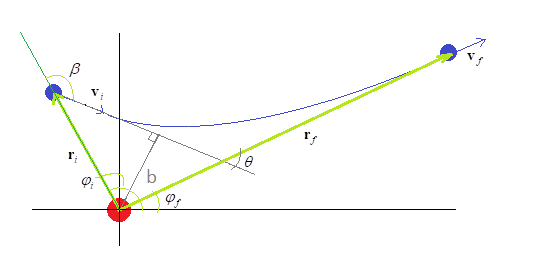
**Two-particle inverse square collisions**

Let’s reconsider collisions now, or, open trajectories if you will. From momentum and energy conservation, we were able to determine everything about the trajectories except the angle of the vector δ**p**. Now that we have all information about the force, we ought to be able to determine that angle. that we the final velocities of the particles if we were supplied the final angle at which the particles scattered. But now that we know the force we can determine the final trajectories w/o any supplemental information. Let’s analyze a collision between two particles experiencing a repulsive inverse square force. Consider a situation as shown below. The center of mass vector R and relative vector r is shown for illustration’s sake. Note that the initial velocities of the two masses aren’t assumed to be parallel; everything is arbitrary.



[I’m using subscript 0 and i synonymously] Think I’ll introduce the impact parameter b, and scattering angle, θ. Could observe, for what it’s worth: (φi – φf) + π – β + θ = π → φf = φi – β + θ. β has to do with the impact parameter, b, and then θ is the scattering angle.



Note the vantage point of our collision here is always particle 2 – before and after the collision. Basically we’re always in particle 2’s reference frame, and so it’s always stationary in this frame. This is a little different than before when we studied generic collisions. There we examined two different special reference frames: the center of mass, w/r to which both particles are moving, and particle 2’s initial reference frame, w/r to which particle 2 is *initially* stationary, but not finally after the collision. Anyway, we’ll suppose that they have masses m1 and m2, and initially have velocities **v**1i and **v**2i when they are far away before the ‘collision’. We want to determine what the final velocities will be **v**1f and **v**2f a long time after the ‘collision’. We have basically already done the work. We just have to translate into what we want. So we need to determine what the velocity of each mass is as a function of time. From our equations we have:



And the equations for **r**(t) and **R**(t) are:



where



and **i**, **j**, are unit vectors in the plane of the motion, as illustrated above. Of course **R**0 and r0, φ0, as well as E and ℓ would be determined from initial conditions. What we are really interested in, however, are the velocities. So taking the derivative of these equations:



and these velocities are:



So if we can figure out what **v**(t) will look like in the long time limit, then we’ll be able to get **v**1 and **v**2 in the long time limit. A long time after the collision, the radial velocity will go to a constant (since radial force goes to zero). Generally it would be difficult to determine what these various terms go to as t → ∞. But recall from conservation of relative angular momentum that,



Filling this in simplifies the velocity to:



Now since r(t→∞) → ∞, we have:



To determine the radial velocity as t → ∞ we can differentiate our formula for t(r) with respect to r, invert it, and obtain:



and in the large time limit, r will go to infinity, which would simplify our expression to:



Note this is nothing other than the conservation of relative energy equation. Now for the angle, in the large time limit (t → ∞), φ will go to a constant, to determine what this constant is we can examine the formula for t(φ),



And recall that this is basically the conservation of relative angular momentum equation. Now as t → ∞, the integral must blow up (so as to equal t, which is ∞). This would only happen if the integrand becomes singular as φ approaches the upper limit of integration. And this will happen iff:



Guess we can write as:



And identify the first angle in parentheses as β and the second as θ. This follows since, refering to the diagram above, if we take r0 → ∞, while keeping b fixed, then β would go to π, and we’d have φ = φ0 + θ – β → φ0 + θ – π. And we see that:



So θ is indeed π – 2cos-1(1/ε), and β is the other guy. So then we have:



And so then the final velocities will be:



If we take the particles to be initially far apart, then we can say:



which reduces this to:



where (β should be obvious from looking at our initial setup though),



Defining θ0v = φ0 – β as the initial angle of the relative velocity vector, we have:



where,



And of course we should compare this to our earlier generic analysis based simply on conservation laws.